

College Entrance Examination Board  
Advanced Placement Examination  
MATHEMATICS: CALCULUS AB  
SECTION II  
Time—1 hour and 30 minutes

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

1. Let  $f(x) = \cos x$  for  $0 \leq x \leq 2\pi$ , and let  $g(x) = \ln x$  for all  $x > 0$ . Let  $S$  be the composition of  $g$  with  $f$ , that is,  $S(x) = g(f(x))$ .
- (a) Find the domain of  $S$ .
  - (b) Find the range of  $S$ .
  - (c) Find the zeros of  $S$ .
  - (d) Find the slope of the line tangent to the graph of  $S$  at  $x = \frac{\pi}{3}$ .

2. Consider the function  $f$  defined by  $f(x) = (x^2 - 1)^3$  for all real numbers  $x$ .
- (a) For what values of  $x$  is the function increasing?
  - (b) Find the  $x$ - and  $y$ -coordinates of the relative maximum and minimum points. Justify your answer.
  - (c) For what values of  $x$  is the graph of  $f$  concave upward?
  - (d) Using the information found in parts (a), (b), and (c), sketch the graph of  $f$  on the axes provided.

3. Given the function  $f$  defined for all real numbers  $x$  by  $f(x) = e^{\frac{x}{2}}$ .
- (a) Find the area of the region  $R$  bounded by the line  $y = e$ , the graph of  $f$ , and the  $Y$ -axis.
  - (b) Find the volume of the solid generated by revolving  $R$ , the region in (a), about the  $X$ -axis.

4. Let  $f$  and  $g$  and their inverses  $f^{-1}$  and  $g^{-1}$  be differentiable functions and let the values of  $f$ ,  $g$ , and the derivatives  $f'$  and  $g'$  at  $x = 1$  and  $x = 2$  be given by the table below:  
Determine the value of each of the following.

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	5	4
2	2	$\pi$	6	7

- (a) The derivative of  $f + g$  at  $x = 2$
  - (b) The derivative of  $fg$  at  $x = 2$
  - (c) The derivative of  $\frac{f}{g}$  at  $x = 2$
  - (d)  $h'(1)$  where  $h(x) = f(g(x))$
  - (e) The derivative of  $g^{-1}$  at  $x = 2$
5. A particle moves along the  $X$ -axis with acceleration given by  $a(t) = 2t - 10 + \frac{12}{t}$  for  $t \geq 1$ .
- (a) Write an expression for the velocity  $v(t)$ , given that  $v(1) = 9$ .
  - (b) For what values of  $t$ ,  $1 \leq t \leq 3$ , is the velocity a maximum? Justify your answer.
  - (c) Write an expression for the position  $x(t)$ , given that  $x(1) = -16$ .

6. A rectangle has a constant area of 200 square meters and its length  $L$  is increasing at the rate of 4 meters per second.
- (a) Find the width  $W$  at the instant the width is decreasing at the rate of 0.5 meters per second.
  - (b) At what rate is the diagonal  $D$  of the rectangle changing at the instant when the width  $W$  is 10 meters?

7. Let  $f$  be the real-valued function defined by  $f(x) = \sin^3 x + \sin^3 |x|$ .
- (a) Find  $f'(x)$  for  $x > 0$ .
  - (b) Find  $f'(x)$  for  $x < 0$ .
  - (c) Determine whether  $f(x)$  is continuous at  $x = 0$ . Justify your answer.
  - (d) Determine whether the derivative of  $f(x)$  exists at  $x = 0$ . Justify your answer.

END OF EXAMINATION